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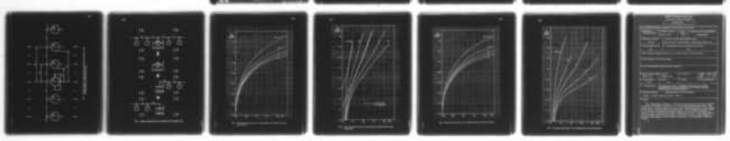

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STRESS INTENSITY FACTORS FOR A CRACKED HOLE IN A ROW OF HOLES

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SUMMARY

The compounding technique of obtaining stress intensity factors is applied to cracks at the edge of one hole in a row of periodically spaced holes. The cracks (one or two) lie along the line of holes and perpendicular to the uniform stress which is applied remote from the holes. This configuration models cracks at rivet holes in airframe structures reinforced with riveted stiffeners, if there is little in-plane load-transfer through the rivets. A particular feature of the solution is the high accuracy of the results at short crack lengths - a necessary requirement for the reliable estimation of the lifetimes of growing cracks.

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LIST OF CONTENTS

1 INTRODUCTION

In engineering materials used in aircraft structures cracks may be present either at manufacture or they may appear during the service life. To ensure safety and to optimise inspection schedules, it is necessary to know both the residual strength of the cracked structure and the rate at which the crack grows when subjected to service loads. Both are dependent on the stress intensity factor which governs the stress field at the tip of the crack.

The stress intensity factor is known for many simple configurations[1-3]. However, in practice, cracks occur in more complex configurations. Most methods for determining stress intensity factors[4], either theoretically or experimentally, are both costly and time-consuming. Hence there is a need for a simple, quick, and easy-to-use method for obtaining stress intensity factors for practical configurations from the known solutions for simple ones. A compounding method was developed by Cartwright and Rooke[5] in which boundary-boundary interactions are not significant. This method has been modified to include cracks in stiffened structures[6] and further modified to include the case of a crack at the edge of a hole[7]. In this latter modification, it was necessary to include a contribution for boundary-boundary interaction.

In this Report the compounding method is applied to the problem of cracks at the edge of a hole which is in a row of holes in a uniformly stressed sheet; the stress which acts in a direction perpendicular to the row of holes causes the cracks to grow along the line of holes. This configuration is representative of some airframe components. For example, cracks may develop at a rivet hole in a longitudinal stiffener in a pressure cabin, in which the major in-plane loading arises from the hoop stress, with little in-plane load transfer through the rivet. An analagous situation may arise in a wing skin in which a chordwise crack may develop at a rib/skin fastener under wing bending fatigue loads.

In order to apply the compounding method to such configurations the stress intensity factors are required for the following ancillary configurations; an isolated hole with one or two cracks (Case 1.3.3, Ref 1) and a crack near an isolated hole (Case 1.3.5, Ref 1). The boundary interaction term is estimated fro[illegible] results for a cracked hole subjected to localized forces on its pe[illegible]. The compounding method gives accurate solutions for short crack-lengths - the stress intensity factor tends to that of a crack at the edge of a hole with a stress *concentration* factor K_t, which is determined by the geometry of the uncracked structure[9,10]. The importance of accuracy for short cracks lies

073 in the fact that a major portion of the structures useful life is spent when

the crack is short. Unless reliable predictions of strength and crack-growth rate of short cracks can be made, safety considerations may result in the use of over-conservative assumptions in design and inspection frequency, with consequent economic penalties.

2 THE COMPOUNDING METHOD

The method of compounding is described in Ref 5 where the resultant normalized stress intensity factor Q_r is found by using ancillary configurations that in combination approximate to the one under consideration. The structural configurations studied in this Report are those of infinite sheets with rows of holes (radius R) in a line, uniformly stressed remote from and perpendicular to the line of holes (see Fig 1). Two configurations are considered; the first where one of the holes is symmetrically cracked and the second where the hole has only one crack. In both cases a is the distance from the centre of the hole to the tip of the crack. In order to calculate the effect of other boundaries on a cracked hole we have to substitute for the cracked hole an equivalent simple crack, which has the same normalized stress intensity factor as the cracked hole; the procedure used is the same as that in Ref 7. In order to obtain an expression for the *equivalent crack length*, we consider the stress intensity factor of a simple crack of length $2a'$ in an infinite sheet under uniform uniaxial tensile stress σ remote from the crack,

$$K = \sigma\sqrt{\pi a'} \tag{1}$$

as a definition of the equivalent crack. This must be the same as that for an isolated cracked hole, which is K_0 and is given by,

$$K_0 = Q_0\sigma\sqrt{\pi a} \quad , \tag{2}$$

where Q_0 is the normalised stress intensity factor of a cracked hole remote from all boundaries. Comparison of equations (1) and (2) gives

$$a' = Q_0^2 a \quad . \tag{3}$$

This means that for the ancillary configurations the cracked hole is replaced by a crack of length $2Q_0^2a$ (see Fig 2).

When the cracked hole is replaced by an equivalent crack of length $2a'$ the distance from the crack-tip to the boundary is kept constant, ie for the right-hand tip (tip A in Fig 2)

$$b'_n - a' = b_n - a \ , \tag{4}$$

where b'_n is the distance from the centre of the equivalent crack to the nth ($n > 0$) boundary on the right and b_n is the distance from the centre of the cracked hole to the nth boundary; similarly for the boundaries on the left ($n < 0$)

$$b'_n + a' = b_n + a \ . \tag{5}$$

The basic compounding formula for a crack in the presence of an infinite array of boundaries, is given by[4],

$$K_r = \bar{K} + \sum_{n=-\infty}^{n=\infty} (K_n - \bar{K}) + K_e \tag{6}$$

where $\bar{K}$ is the stress intensity factor of a crack of length $2a$ in an infinite sheet subjected, remote from the crack, to a uniform uniaxial tensile stress σ and $\bar{K}$ is given by

$$\bar{K} = \sigma\sqrt{\pi a} \ . \tag{7}$$

K_n is the stress intensity factor of the crack in the presence of the nth boundary only and K_e is the contribution due to boundary-boundary interactions.

Equation (6) can be written in terms of the normalized stress intensity factor $Q(= K/\bar{K})$ as

$$Q_r = 1 + \sum_{n=-\infty}^{n=\infty} (Q_n - 1) + Q_e \ . \tag{8}$$

The introduction of the equivalent crack-length a' into the ancillary configurations modifies[6] equation (8) to

$$Q_r = Q_0 \left[1 + \sum_{n=-\infty}^{n=\infty} (Q_n' - 1) \right] + Q_e \quad , \qquad n \neq 0 \quad , \tag{9}$$

where the normalized stress intensity factors are Q_0 for an isolated cracked hole and Q_n' for the equivalent crack in the presence of the nth boundary only.

A general expression for the correction Q_e has been derived[5] using an alternating technique; it is not, however, easy to calculate. It has been suggested[7] that an approximation for Q_e may be obtained by considering the total effect to be that of two point forces P applied to the top and bottom of the cracked hole. The magnitude of the forces P is determined so as to give the same stress *concentration* factor K_t at the edge of the uncracked hole as that which occurs due to all the other boundaries in the configuration. These forces P are a function of the individual configuration through its K_t which is available for many common configurations[9,10]. Values of Q_e/Δ , for a hole with one or two cracks as a function of the crack length divided by the hole radius, are given in Ref 7, where

$$\Delta = \frac{2P}{\pi R \sigma} = \left(K_t - 3 \left(1 + \sum_{n=1}^{N} (M_n' - 1) \right) \right) , \tag{10}$$

where M_n' is the normalized stress intensity factor as $a' \to 0$ in the presence of the nth boundary only.

The complete compounding formula is therefore;

$$Q_r = Q_0 \left[1 + \sum_{n=1}^{N} (Q_n' - 1) \right] + \left[K_t - 3 \left(1 + \sum_{n=1}^{N} (M_n' - 1) \right) \right] \left(\frac{Q_e}{\Delta} \right) . \tag{11}$$

2.1 <u>Application to a symmetrically cracked hole</u>

The geometry of the configuration is defined by the hole radius R , the distance a between the hole-centre and the crack-tip, and the spacing b between hole-centres.

Graphs of Q_0 and Q_e/Δ for a single hole with two equal-length cracks are given as a function of a/R in Ref 7. Values of Q_n' for the ancillary configuration of a crack located near a hole can be obtained from

Case 1.3.5, Ref 1. Q'_n is given as a function of a'/d'_n and d'_n/R where d'_n is the distance from the centre of the equivalent crack to the edge of the nth hole in the ancillary configuration ($d'_n = b'_n - R$). The dependence of a'/d'_n and d'_n/R on a/R and b/R are given in the Appendix. The first term in equation (11) can be evaluated from known values of Q_0 and Q'_n ; the value of $(Q'_n - 1)$ was negligible for $|n| > 2$ for all values of a/R and b/R .

Values of K_t and M'_n are required in order to calculate the boundary interaction term Q_e . The values of K_t for various b/R ratios are given[9,10] in Table 1.

Table 1

Stress *concentration* factors for a hole in a periodic row of holes

b/R	3.0	3.5	4.0	5.0	10
K_t	3.92	3.44	3.24	3.10	3.01

The values of M'_n are obtained from

$$M'_n = \lim_{a' \to 0} \{Q'_n\} ; \qquad (12)$$

the appropriate values of a'/d'_n and d'_n/R are given in the Appendix. For the configurations considered here the boundary interaction term Q_e was small ($Q_e < 5\%\ Q_r$).

Curves of $K_r/(\sigma\sqrt{\pi R})$, ie $\sqrt{a/R}\, Q_r$, are plotted as a function of a/R , in Figs 3 and 4, for various values of b/R. Fig 3 contains the results for short cracks $1.0 \leqslant a/R \leqslant 1.25$ and Fig 5 for longer cracks. In Fig 4 the stress intensity factor tends to that for a line crack of length $2a$ located symmetrically in an array of holes; the solution for this configuration may be obtained by compounding from Case 1.3.7, Ref 1. Two curves are shown for this approximation ($b/R = 3$ and 4) in Fig 4.

2.2 Application to a hole with one crack

The geometry of the configuration is defined, as before, by the hole radius R , the distance a from the hole-centre to the crack-tip and the spacing b of the holes.

The functions Q_0 and Q_e/Δ which are needed to calculate Q_r from equation (11) are available[1], as a function of a/R. The ancillary configuration is again the equivalent crack in a row of holes (Fig 1); the parameters a'/d'_n and d'_n/R which are required to obtain Q'_n (Case 1.3.5, Ref 1) are given in the Appendix. Again $(Q'_n - 1)$ was negligible for $|n| > 2$. Since K_t and M'_n are evaluated for zero crack-length, they are the same as in the two-crack problem.

The resultant normalized stress intensity factor Q_r can now be compounded from equation (11); the boundary interaction term was again small ($Q_e < 5\% \ Q_r$). Curves of $K_r/(\sigma\sqrt{\pi R})$, ie $\sqrt{a/R}\ Q_r$, are plotted in Figs 5 and 6 as a function of a/R for various values of b/R. Fig 5 contains the results for short cracks $1.0 \leqslant a/R \leqslant 1.25$ and Fig 6 for longer cracks. For the longer cracks the stress intensity factor will tend to that for a line crack of length $a + R$ located eccentrically in an array of holes; the solution for this configuration may be obtained by compounding from Case 1.3.5, Ref 1.

3 DISCUSSION

In this Report the compounding technique has been used to calculate the stress intensity factor for a cracked hole in a row of equally spaced holes. The technique is not limited to the configurations considered, ie one crack or two equal-length cracks at the edge of one of the holes. The more general problem of a hole with two unequal-length cracks in a non-periodic row of dissimilar holes can be solved using the same ancillary configurations[1,7]. The presence of the holes near the cracked hole increases the stress intensity factor over that of an isolated cracked hole; the effect is negligible if the holes are more than 10R apart, and increases as the hole-spacing decreases (see Figs 3 to 6). In all cases the effect of the hole nearest to the tip being considered is about ten times greater than the effect of the next nearest hole on the same side of the tip; the effect of all the other holes on the same side of the tip is negligible ($\lesssim 1\%$). The holes on the far side of the crack from the tip being considered have a smaller effect (20 to 25%) than those on the near side.

A particular advantage of the compounding technique for cracked-hole problems is that the stress intensity factor is determined accurately for short cracks. This accuracy is ensured by relating the boundary-boundary interactions to the stress *concentration* factor K_t in the uncracked configuration. For long cracks the solutions obtained approach those obtained by replacing the hole plus crack(s) by a simple crack of the same overall length as the hole plus crack(s). It is estimated that the maximum errors in the stress intensity

factors are $\lesssim 10\%$ overall, and much less for the important region of short cracks; it is the behaviour of short cracks which dominates some damage-tolerant design calculations since a large proportion of the useful life of a cracked component is spent while the crack is short.

Although the boundary-boundary interaction term Q_e was included in order to ensure accuracy for short cracks the contribution to the stress intensity factor was small ($\lesssim 5\%$) for the configurations considered. A more closely spaced array of holes would result in larger values of Q_e .

Comparison of Figs 3 and 4 with Figs 5 and 6 show that for the same values of b/R and a/R the stress intensity factors for one crack are always less than for two cracks; for short cracks ($a/R < 1.25$) the difference is less than 5%.

4 CONCLUSIONS

(1) The compounding technique can be applied to a cracked hole in a row of holes to give stress intensity factors for a common airframe configuration.

(2) The solutions are acceptable for most engineering applications; they are particularly accurate for small cracks, a necessary condition for the use of the method in damage tolerant designs.

Appendix

DETERMINATION OF PARAMETERS FOR ANCILLARY CONFIGURATIONS
(See section 2)

The parameters a'/d'_n and d'_n/R required in the ancillary configuration must be expressed in terms of a/R and b/R. The condition that the distance from the crack-tip to the boundary should be the same in the ancillary configuration as in the original configuration led to equations (4) and (5) which can be written in terms of d the distance from the crack centre to the boundary edge:

$$\left.\begin{aligned} d'_n &= d_n - a + a' , \quad n > 0 \\ \text{and} \quad d'_n &= d_n + a - a' , \quad n < 0 . \end{aligned}\right\} \qquad \text{(A-1)}$$

Since

$$d_n = |n|b - R , \qquad \text{(A-2)}$$

equation (A-1) becomes

$$\left.\begin{aligned} \frac{d'_n}{R} &= \frac{nb}{R} - 1 + \frac{a}{R}(Q_0^2 - 1) , \quad n > 0 \\ \text{and} \quad \frac{d'_n}{R} &= \frac{|n|b}{R} - 1 - \frac{a}{R}(Q_0^2 - 1) , \quad n < 0 . \end{aligned}\right\} \qquad \text{(A-3)}$$

Therefore, since $a' = Q_0^2 a$,

$$\frac{a'}{d'_n} = \frac{Q_0^2 \frac{a}{R}}{d'_n/R} . \qquad \text{(A-4)}$$

For the special case $a' = 0$, required in the calculation of M'_n , the parameter d'_n/R have the following values:

$$d'_n/R = \frac{nb}{R} - 2 , \text{ for } n > 0 , \qquad \text{and } d'_n/R = \frac{nb}{R} , \text{ for } n < 0 .$$

.... (A-5)

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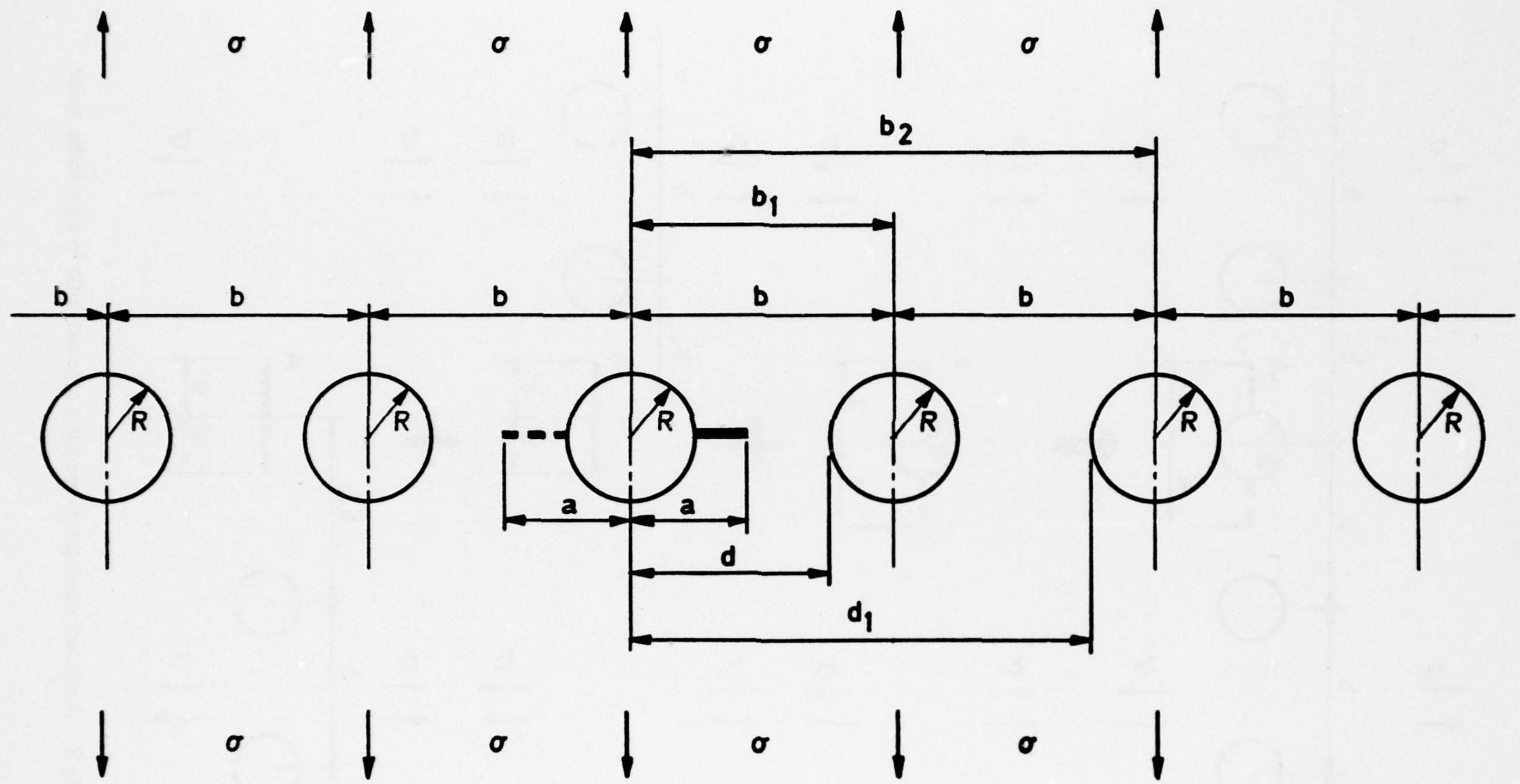

Fig 1 Configuration for a hole with one or two cracks at its edge, in a line of holes, subject to remote uniaxial stress

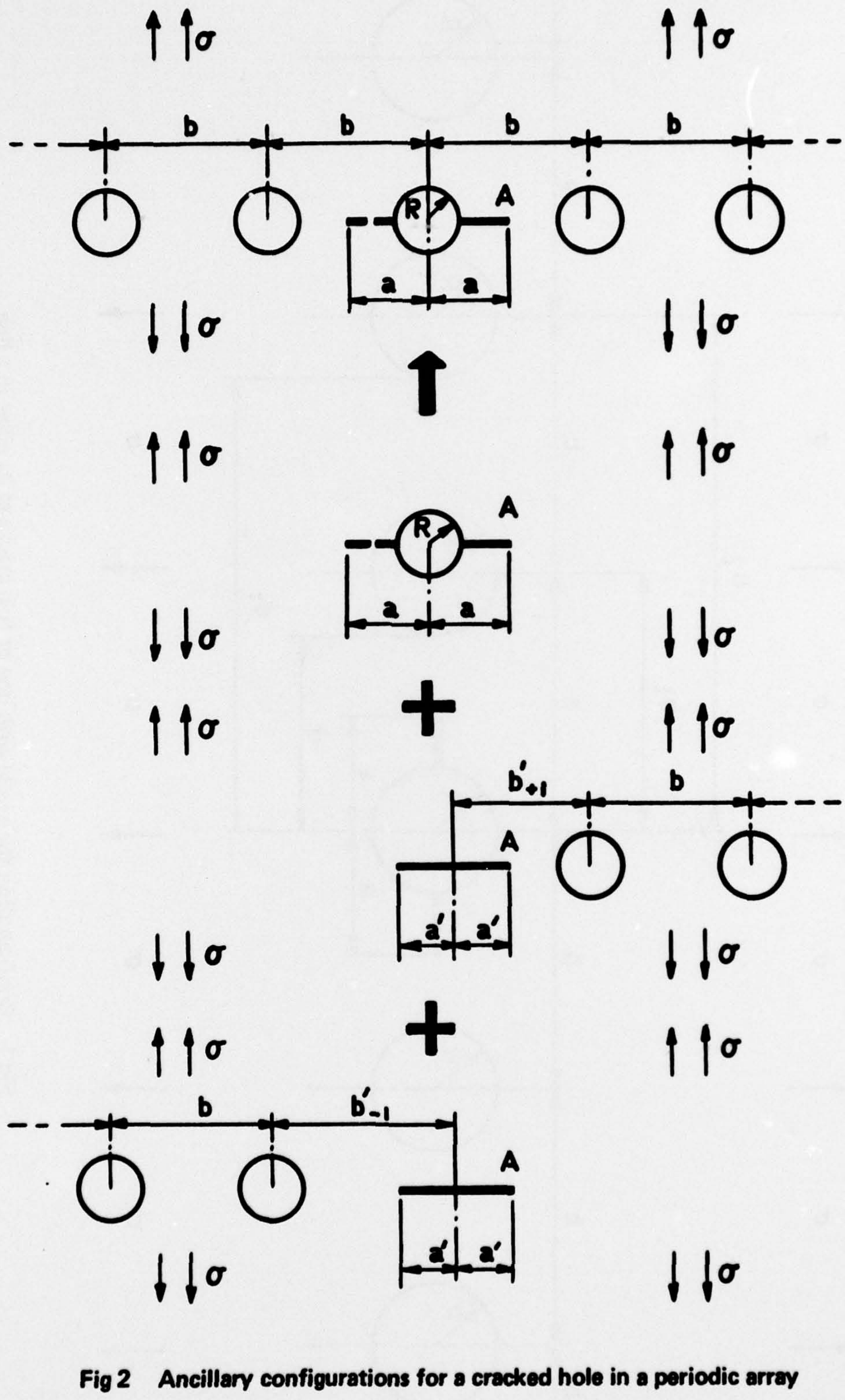

Fig 2 Ancillary configurations for a cracked hole in a periodic array

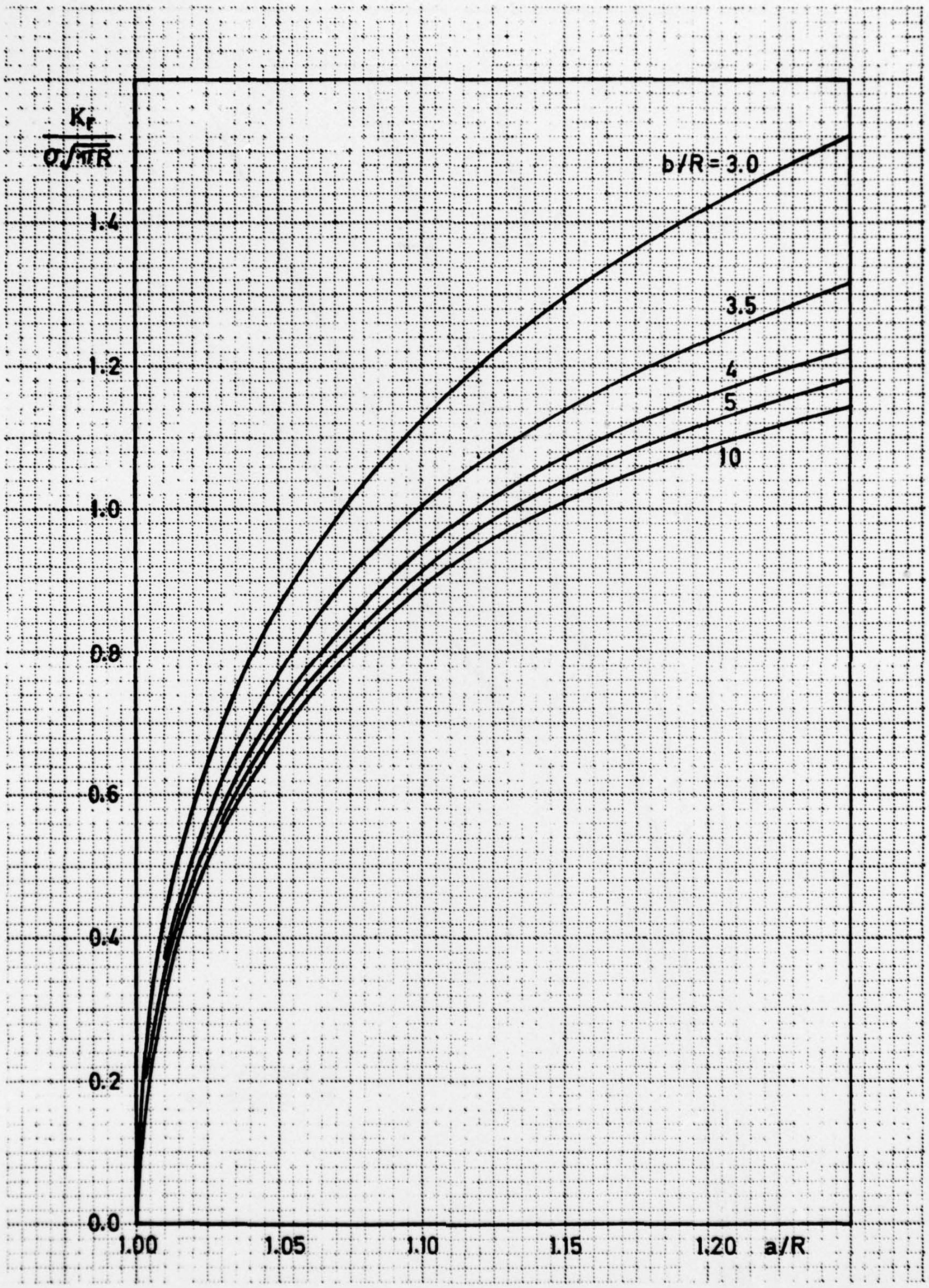

Fig 3 The stress intensity factor for a symmetrically cracked hole in an array (short cracks)

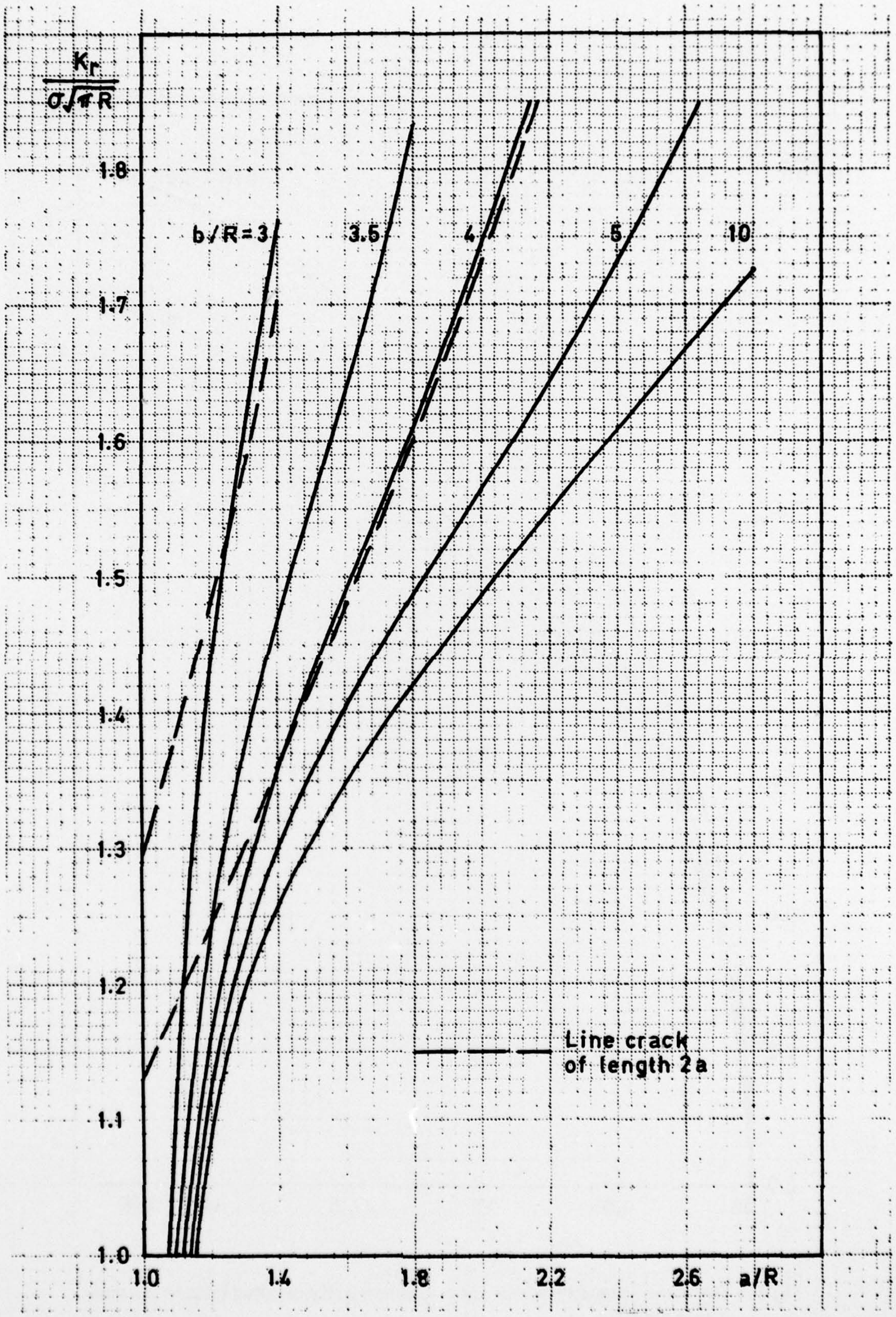

Fig 4 The stress intensity factor for a symmetrically cracked hole in an array (long cracks)

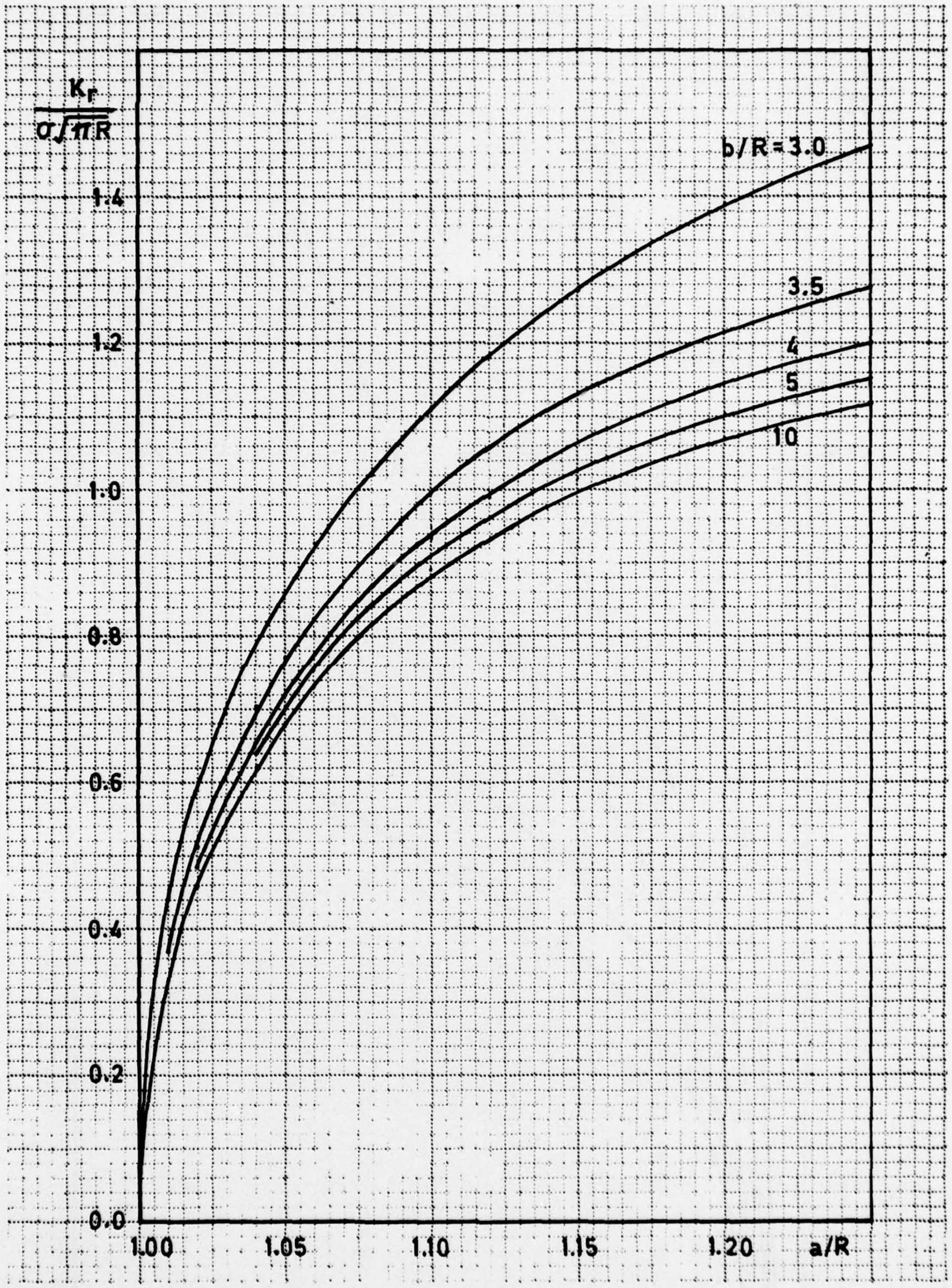

Fig 5 The stress intensity factor for a cracked hole in an array (short cracks)

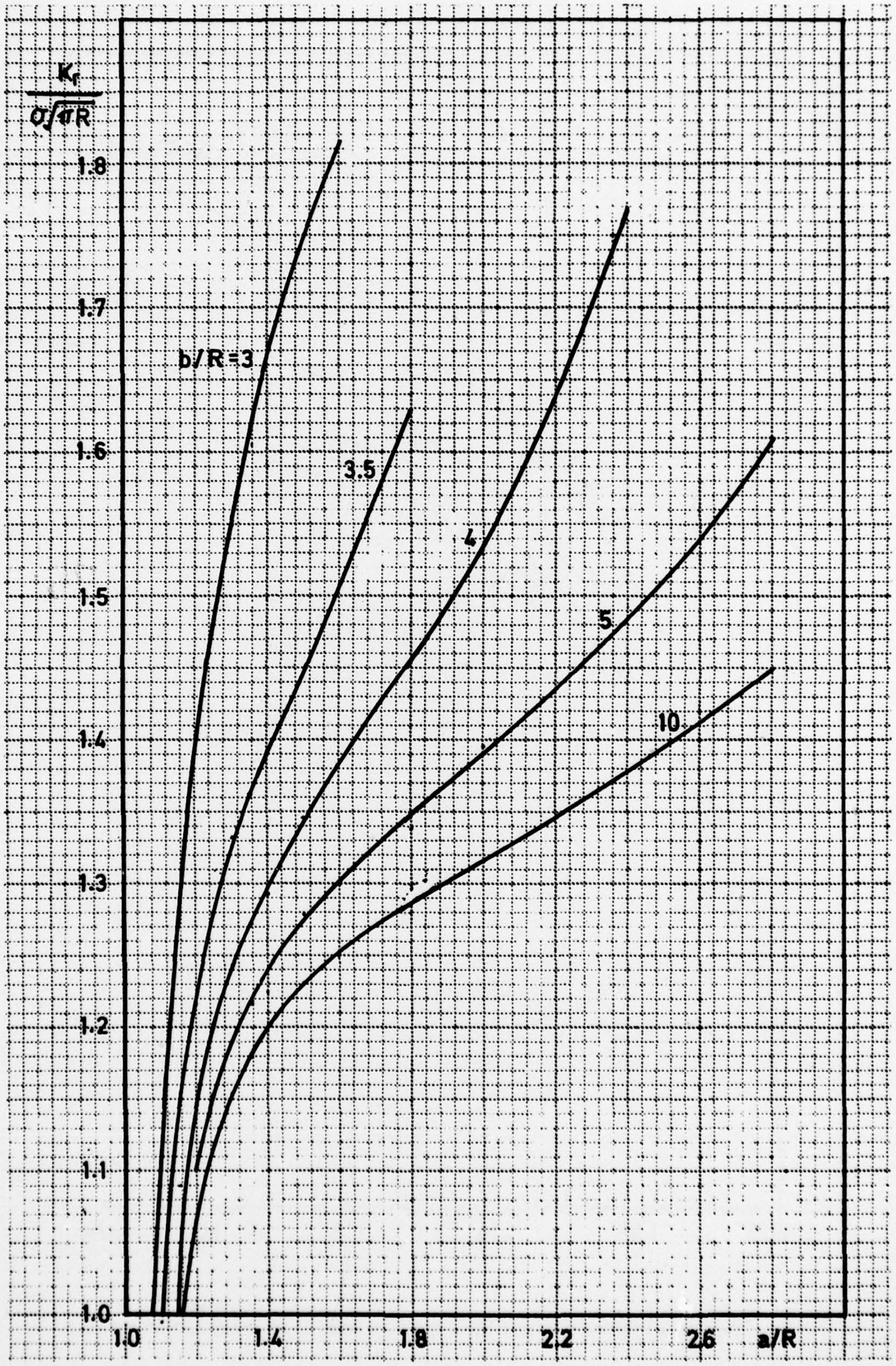

Fig 6 The stress intensity factor for a cracked hole in an array (long cracks)

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17. Abstract

The compounding technique of obtaining stress intensity factors is applied to cracks at the edge of one hole in a row of periodically spaced holes. The cracks (one or two) lie along the line of holes and perpendicular to the uniform stress which is applied remote from the holes. This configuration models cracks at rivet holes in airframe structures reinforced with riveted stiffeners, if there is little in-plane load-transfer through the rivets. A particular feature of the solution is the high accuracy of the results at short crack lengths - a necessary requirement for the reliable estimation of the lifetimes of growing cracks.

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